Price as a Signal of Product Availability: Is it Cheap?

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Firms have more information than their customers about the availability of the product. Given the increase in the strategic nature of customers, who consider both prices with future availability risk in making their purchase decisions, the firm may be interested in communicating this information to customers to impact the timing of their purchase. In this paper, we study whether firms can indeed use prices to signal product availability information to their customers. We prove that a signaling or separating equilibrium always exists in which prices perfectly communicate the product availability. Further, we find conditions on the model primitives for which separation is costly, in particular, the firm must price differently from its full information price to signal its product availability.

1. Introduction

Firms selling products typically have more information than their customers about the availability of the product. This additional information can stem from knowledge of their inventory or better understanding of the overall demand, or both. Given the increase in the strategic nature of customers, who consider both prices with future availability risk in making their purchase decisions, the firm would like to be able to communicate this information to customers to impact the timing of their purchase. One means of communicating this information could be to announce it to the customers; however, such announcements are not credible, since the firm would always want to indicate low future availability. Another possibility is to use prices as a signaling device. This mode of communication is especially important because most customers base their purchase decisions solely on the product price. Our main research question is whether firms can indeed use prices to signal product availability information to their customers.

While a priori using prices to communicate seems quite a tempting proposition, there are trade-offs associated with it. In particular, consider a firm with certain product availability. This firm may choose to post lower than usual prices to ensure that a larger population is interested in buying the product. Yet, if it lowers the price too much, it may give customers the indication of ample supply and thus may reduce the urgency to buy early. Thus, in deciding its price signal the firm must consider the trade-off between communicating information via prices and its consequent effect on revenues. We would like to highlight that extant literature has studied prices both as a means of regulating demand to maximize revenues and as a means of signaling product quality. This paper is, to the best of our knowledge, the first that considers price as a signal of product availability.

Formally, we consider the pricing problem that a firm faces when it sells an initial inventory to customers over two periods without replenishments. The first period customers are of two types: strategic customers and myopic customers. The strategic customers can purchase in either period and make the decision to purchase by maximizing their expected net value. In particular, they consider the trade-off between the price risk and the availability risk. On the other hand, the myopic customers purchase if their valuation exceeds the current price and leave the system otherwise (they do not wait for the second period). The initial inventory and the demand distributions are the firm's private information and the customers share a common belief on these quantities.

Our goal is to understand if prices can be used by the firm to signal or communicate its private information, and how expensive would doing so be. To this end, we find the existence of pooling and separating equilibria. The pooling equilibria are non-communicating in the sense that the firm sets the same price irrespective of its private information. Thus, the prices are uninformative of the firm's private information and the strategic customers decide when to purchase based on their prior beliefs. The separating equilibria are communicating in the sense that prices communicate the firm's private information perfectly. We further categorize these separating equilibria as: (i) Costless separating and (ii) Costly separating equilibria. In the costless separating equilibria, the firm prices at its full information prices in each information state, and hence pricing communicates the firm's private information to the customers with the firm achieving its full information revenues.

On the other hand in the costly separating equilibrium, the firm prices differently from that under full information. In particular, if there are two firm types or states of the world, then in a costly separating equilibrium, in the higher availability state, the firm prices at its full information level, whereas in the lower availability state, the firm prices higher than that under full information to signal the availability risk to the customers. This price increase lowers the firm's revenue as compared to the full information solution in the low availability state, and thus is costly. This drop in firm's revenues leads to an increase in the overall customer surplus, and can be interpreted as the customers extracting rents from the firm for the communication of information.

We prove that a signaling or separating equilibrium always exists in which prices perfectly communicate the product availability. Further, we find conditions on the model primitives for which separation is costly. It turns out that, loosely speaking, if the likelihood of a stock-out is similar in the two state of world, then we obtain costly separation. We also study the presence of costless separation as a function of the number of myopic customers. Intuitively, one expects an increase in myopic customers to shift the firm's focus away from communicating availability information, and thus facilitate costless separation. Interestingly, we find that the presence of the costless separating equilibrium is non-monotone in the number of myopic customers, that is, as the difference in product availability between the two states increases, the number of myopic customers needed to ensure costless separation need not decrease.

To summarize, extant literature studies prices largely as a device to shape demand and increase revenues. In this paper, we consider the use of prices as a product availability signaling device in addition to these roles. We find that there are cases when the firm can perfectly communicate this private information to customers in a costless fashion (compared to its full information solution). Interestingly, we also find cases in which the firm must forgo some revenue to communicate this information in a costly fashion.

2. Literature Review

There has been a recent spurt of interest in incorporating strategic consumer behavior in pricing models. One of the earliest papers on this topic is that of Besanko and Winston (1990), which uses a deterministic model to study a firm's dynamic pricing decisions and demonstrates a price-skimming effect. Su (2007) also considers a deterministic model in which customers differ in their valuations and patience levels and decide when to purchase, and shows that in such a setting, both markups and markdowns are possible. Similar pricing trajectories are observed in Xu and Hopp (2004), which studies a continuous-time model in which customers arrive according to a Poisson process. In that paper, the nature of the price trajectory is shown to depend on the customers' price sensitivity. Aviv and Pazgal (2008) studies a similar setting in the context of a seasonal product for which the customers' valuations decrease with time. They show that while the presence of strategic customers suppresses the benefits of price discrimination, they identify settings in which price discrimination can still be used effectively. Recently, Levin, McGill and Nediak (2010) shows that the impact of strategic customer behavior can be mitigated by a firm by combining capacity selection with an appropriate pricing policy.

We use a two period model with myopic customers as in Cachon and Swinney (2009). There, the authors investigate the value of "quick response" in the presence of strategic customers and find that quick response offers more value with strategic customers. While the underlying set up is similar, our research question is fundamentally different. We seek to investigate the role of prices in signaling the firm's private information on availability to the customers. In Cachon and Swinney (2009), while availability is not observable, the firm possesses no private information, i.e., all parameters used by the firm to make its decision are public, and hence can be anticipated by the customers.

This issue of inducing strategic customers to purchase early has been studied in various different aspects. For instance, Liu and van Ryzin (2008) demonstrates how quantity decisions alone at fixed price levels can be used to induce strategic customers to purchase early. Focusing on product availability, Zhang and Cooper (2008) studies the value of restricting product availability in the second period, and they find conditions when such an approach may improve revenues and when it may not. Su and Zhang (2009) is another paper that studies the customers trade-offs between buying early for sure and later with limited available under fixed price levels. The paper first establishes the equilibrium when customer observe the inventory levels, and study models when this information is not observable and instead the firm offers availability guarantees to customers. The customer's decision to postpone purchases is studied in a slightly different context in Caldentey and Vulcano (2007), in which strategic customers decide between purchasing immediately or can attempt to buy later by entering an auction.

Dana Jr (2001) is a related paper that studies competition in price and availability between multiple firms. As in our paper, the availability of the firm is not observable and is inferred from posted prices. However, a major difference is that in Dana Jr (2001), the customers decide which firm to visit based on the posted prices and in our paper, the customers decide whether to purchase immediately or postpone their purchase based on the posted price. So, in that paper, a customer may be lost by the firm if the customer anticipates a low availability, whereas in our paper, the anticipation of low availability in fact makes the customer want to purchase earlier, at a higher price. The main result differs accordingly, while in that paper, higher prices signal higher availability, in our paper, higher prices in fact signal scarcity.

The literature on signaling models in operations management is quite scant. One of the first papers to study such a model was Cachon and Lariviere (2001),which studies information sharing in a supply chain. More recent papers that study signaling models in such a context are Akan, Ata and Lariviere (2011), Anand and Goyal (2009), Debo and Van Ryzin (2009), Allon and Bassamboo (2011). A closely related paper is Gaur, Lai, Raman, Schmidt and School (2011), which studies a newsvendor setting in which a firm signals its private demand information to investors by selecting quantity. They find instances in which a firm may be unable to signal its higher demand, and consequently under-invests in quantity. However, in our paper, the firm is always able to communicate its private information; in some cases it is costless, and in others costly. Debo and Veeraraghavan (2010) is also related to this paper. In that paper, the authors study prices and congestion as a signal of quality. (Prices as a signal of quality in general economic models have been studied in Milgrom and Roberts (1986) and Bagwell and Riordan (1991).) While not signaling models per se, Yin, Aviv, Pazgal and Tang (2009) and Dasu and Tong (2010) look at the impact of sharing information with customers. Yin et al. (2009) find that it may be more profitable for the firm to display a single item of the product, when it is available, rather than display the entire inventory. Dasu and Tong (2010) studies the impact of posting prices and show that such schemes can perform nearly optimally.

3. Model

The focus of this paper is the ability of a firm to use pricing to signal availability risk to its customers. In particular, we consider a setting in which the firm has private information about its inventory level and/or the demand for its products. For ease of exposition, we focus on a model in which the inventory level held by the firm is its private information whereas the demand is common knowledge. We later show how the analysis of this model applies to models in which this inventory level is determined as a decision by the firm and those in which the inventory level is common knowledge, but the demand is private information (see Section 6 for details). In all of these cases, they key two features are: (i) the firm has private information on the likelihood of a stockout, and (ii) the customers infer the stockout likelihood from the prices set by the firm.[1]

We consider a two period model in which customers arrive in each period. The customers that arrive in the first period can be one of two types: strategic or myopic. The myopic customers are only interested in purchasing the product in the first period and thus do not wait for the second period. Thus, they base their purchase decisions on the first period price alone. We denote the myopic demand in period 1 by D_m. The strategic customers consider both the future availability and prices in making their purchase decision. The strategic customers do not incur any costs in waiting for the next period. We denote the strategic customer demand in period 1 by D_s. The customer demand in period 2 is denoted by D_2. Customer valuations are distributed on $[\underline{v}, \bar{v}]$ and are independent, and identically distributed across all customers (myopic and strategic); we use F to denote the corresponding cumulative distribution function.

We assume that the firm starts with a fixed inventory that is not replenished over the time horizon under consideration. For simplicity, we consider the case in which the firm's initial inventory level can take two values: Q_h, in which case we refer to the firm as an h-type, or Q_l, in which case we refer to the firm as an l-type, where $Q_h > Q_l$. This inventory level is the firm's private information,

[1] The main goal of the paper is to study the ability to signal availability risk using prices, and so we focus on situations in which stockouts may occur in the second period at least for some demand realizations and states of the world.

and not observable by the customers. The customers have an initial prior $\bar{\theta}$ on the firm type being h. The goal of the firm is to maximize its overall revenue over the two periods by setting the prices. One may consider two settings in which the firm has private information regarding the availability risk: one in which the firm has private information on the inventory level, and one in which it has private information on the period 1 demand realization. We initially study the setting in which the period 1 demand realization is known, and customers have beliefs on the initial inventory level. In Section 6 we discuss the opposite situation in which customer have beliefs on the period 1 demand realization, yet the inventory level is common knowledge. Note that the firm and the customers only know the distribution of the period 2 demand. For convenience, we assume that the period 2 price is set at $s = \underline{v}$. We discuss the impact of pricing in both periods in Section 6.

Sequence of events. The firm first observes its type, and then sets the period 1 price. This is followed by the arrival of period 1 demand, which consists of both myopic and strategic customers. Myopic customers purchase if their valuation exceeds the price, whereas strategic customers decide to purchase immediately or wait for period 2. Based on the customers' decisions, period 1 sales are completed. Next, the price is lowered to the sales price, which is followed by the arrival of period 2 demand. Finally, period 2 sales are made to the customers that arrive in period 2 and the left-over strategic customers from period 1. If the demand in any period exceeds the quantity available in that period, then the units are allocated randomly to the customers, so that each customer has the same likelihood of obtaining the product regardless of whether they are strategic, myopic, arrived earlier, etc.

4. Prices can always signal availability

In order to analyze the equilibria of this game, we first characterize the customer's decisions based on the period 1 price set by the firm. Myopic customers purchase in period 1 if the price is less than their valuation. Strategic customers purchase in period 1 only if their net value from purchase exceeds the expected net value they obtain from waiting to purchase in period 2. The indifferent customer's valuation v is characterized by the following relation: his net value from an immediate purchase $(v - p)$ must equal his expected net value from waiting to purchase in period 2, which is equal to $(v - s)$ times the probability of the item being available in period 2. That is, we have

$$(v-p)=(v-s)\mathcal{A}(v,p,\theta), \tag{1}$$

where $\mathcal{A}(v,p,\theta)=\theta(p)A(v,p,h)+(1-\theta(p))A(v,p,l)$ denotes the expected availability in period 2 given the period 1 price of p and belief $\theta(\cdot)$ and for $t=h,l$, $A(v,p,t)$ denotes the expected period 2 availability for type-t firm, i.e.,

$$A(v,p,t):=\mathbb{E}\left[\min\left(\left(\frac{Q_t-[D_s\bar{F}(v)+D_m\bar{F}(p)]}{D_sF(v)+D_2}\right)^+,1\right)\right].$$

Note that in our model, we assume that the firm does not withhold inventory from the customers.

We denote the smallest solution to (1) by $m(p,\theta)$. If there is no solution to (1), which implies that all customers prefer to wait for the second period, then we set $m(p,\theta):=\infty$ to indicate that there is no indifferent customer.[2]

Thus, given a period 1 price of p and belief θ, the total customer demand in period 1 equals

$$d_1(p,\theta):=D_s\bar{F}(m(p,\theta))+D_m\bar{F}(p)$$

and that in period 2 equals

$$d_2(p,\theta):=D_sF(m(p,\theta))+D_2.$$

The firm's optimization. The firm selects a period 1 price in order to optimize its overall expected revenue in light of the customer's beliefs θ. That is, the type-t firm solves

$$\max_{p\geq 0} R_t(p,\theta):=p\min(Q_t,d_1(p,\theta))+s\min((Q_t-d_1(p,\theta))^+,d_2(p,\theta)). \tag{2}$$

Let $R_t^*(\theta)$ denote the optimal value of the above optimization and $p_t^*(\theta)$ its optimizer. We further define p_l^{fi} and p_h^{fi} as the prices charged by the firm under full information when its type is l and h, respectively. In particular $p_l^{fi}=p_l^*(\theta_0)$ and $p_h^{fi}=p_h^*(\theta_1)$, where $\theta_0(p)=0$ and $\theta_1(p)=1$ for all $p\geq 0$. For convenience, we make the following assumption.

Assumption 1. *We assume that under full information, both the potential period 1 revenue and the total revenue of the firm are quasi-concave in the period 1 price for any initial quantity level. That is, for any quantity Q, if $m(p,Q)$ denotes the solution to* (1)*, then $(D_s\bar{F}(m(p,Q))+D_m\bar{F}(p))(p-s)$ and $(D_s\bar{F}(m(p,Q))+D_m\bar{F}(p))(p-s)+s(D_s+D_m+D_2\bar{F}(p))$ are quasi-concave in p.*

[2] Note that if all customers purchase in the first period, then it must be the case that $p=\underline{v}$ and (1) holds with $m(p,\theta)=\underline{v}$.

4.1. Separating equilibrium

A separating equilibrium is characterized by a pair of prices (p_l, p_h) that are selected by the l- and h-type firms respectively in equilibrium and beliefs $\theta_s(\cdot) \in [0,1]$ with $\theta_s(p_h) = 1$ and $\theta_s(p_l) = 0$, such that neither firm-type has any incentive to deviate. That is, for all prices $p \in [\underline{v}, \bar{v}]$, we must have:

$$R_h(p, \theta_s) \leq R_h(p_h, \theta_s) \tag{3}$$

$$R_l(p, \theta_s) \leq R_l(p_l, \theta_s). \tag{4}$$

Further, if such a separating equilibrium exists with both firm types setting their period 1 prices at the full information levels, i.e., (p_l^{fi}, p_h^{fi}), then we call it a costless separating equilibrium, and otherwise we call it a costly separating equilibrium.

Proposition 1. *There always exists a separating equilibrium in pure strategies that survives the intuitive criterion. In this equilibrium, the h-type firm prices prices at its full information level, whereas the l-type firm may or may not do so.*

Proof. Let p_h^{fi} and p_l^{fi} denote the optimal period one prices under full information for the h-type and l-type firm, respectively. Let $K(Q,p) := \mathbb{E}[\min(D_s + D_2 + D_m\bar{F}(p), Q)]$ be the expected total quantity sold over both periods when the period 1 price is p and the total quantity is Q. We can write the expected revenue generated by type-t firm at a period 1 price of p when the customer beliefs are $\theta(\cdot)$ as $R_t^1(p,\theta) + K(Q_t, p)$, where

$$R_t^1(p,\theta) := \min(D_s\bar{F}(m(p,\theta(p))) + D_m\bar{F}(p), Q_t)(p - s).$$

Define $p_l' := \min\{p : p \geq p_l^{fi}, R_h^1(p,l) + sK(Q_h,p) \leq R_h^{fi}\}$. We prove that (p_h, p_l') along with the beliefs $\theta(p) = 1$ for $p < p_l'$ and $\theta(p) = 0$ for $p \geq p_l'$ is an equilibrium, and so if $p_l' = p_l^{fi}$, we obtain costless separation.

Note that using Assumption 1 along with Lemma 1 (which is stated in the Appendix), we obtain $R_h^1(p,l) + sK(Q_h,p)$, $R_h^1(p,h) + sK(Q_h,p)$, and $R_l^1(p,l) + sK(Q_l,p)$ are quasi-concave in the period 1 price p.

Note that the definition of p_l' and the fact that $R_h^1(p,h)+sK(Q_h,p)$ is quasi-concave in p imply that the h-type firm (weakly) prefers to price at p_h^{fi} compared to any price $p_h^{fi}<p\leq p_l'$. That is,

$$R_h^1(p,h)+sK(Q_h,p)\leq R_h^{fi}=R_h^1(p_l',l)+sK(Q_h,p_l') \tag{5}$$

$$\Rightarrow s[K(Q_h,p)-K(Q_h,p_l')]\leq R_h^1(p_l',l)-R_h^1(p,h). \tag{6}$$

Note that $K(Q,\cdot)$ is a decreasing function.

To complete the proof, we need to prove that l-type firm prefers to price at p_l' compared to any other price. Noting that $R_l^1(p,l)+sK(Q_l,p)$ is quasi-concave in p, it follows that the l-type firm will not deviate to a price higher than p_l'. So, we need to show that for $p<p_l'$, we have

$$R_l^1(p,h)+sK(Q_l,p)\leq R_l^1(p_l',l)+sK(Q_l,p_l') \tag{7}$$

$$\Rightarrow s[K(Q_l,p)-K(Q_l,p_l')]\leq R_l^1(p_l',l)-R_l^1(p,h). \tag{8}$$

Note that $R_l^1(p,h)\leq R_h^1(p,h)$ because $Q_l<Q_h$ and $R_l^1(p_l',l)=R_h^1(p_l',l)$ because we must have $D_s\bar{F}(m(p_l^{fi},\theta(p_l^{fi})))+D_m\bar{F}(p_l^{fi})\leq Q_l<Q_h$ and $p_l'\geq p_l^{fi}$. Then, using (6), we can show that (8) holds if

$$K(Q_l,p)-K(Q_l,p_l')\leq K(Q_h,p)-K(Q_h,p_l').$$

This relation holds if the following is true

$$\begin{aligned}
&\min(Q_l,D_s+D_2+D_m\bar{F}(p))-\min(Q_l,D_s+D_2+D_m\bar{F}(p_l'))\\
&\leq \min(Q_h,D_s+D_2+D_m\bar{F}(p))-\min(Q_h,D_s+D_2+D_m\bar{F}(p_l'))\\
\Rightarrow &\min(Q_h,D_s+D_2+D_m\bar{F}(p_l'))-\min(Q_l,D_s+D_2+D_m\bar{F}(p_l'))\\
&\leq \min(Q_h,D_s+D_2+D_m\bar{F}(p))-\min(Q_l,D_s+D_2+D_m\bar{F}(p)),
\end{aligned}$$

which holds because $\min(a,x)-\min(b,x)\leq\min(a,y)-\min(b,y)$ for $a\geq b$ and $x<y$. To see this, consider the case in which $x\leq b$. In this case, the left hand side is zero so the result holds. If $x\in(b,a)$, then left hand side is $x-b$ and right hand side is $\min(a,y)-b$ with $a,y\geq x$, so the result holds. Finally, if $x\geq a$, then left hand side and right hand side equal $a-b$.

Finally, noting that $R_l^1(p,l)+sK(Q_l,p)$ and $R_h^1(p,l)+sK(Q_h,p)$ are quasi-concave in p, it follows that that this equilibrium survives the intuitive criterion. *Q.E.D.*

Proposition 1 proves that even though the firm has private information about its quantity that is not known to the customers, it can *perfectly* signal its quantity, and thus the implied availability, to its customers by pricing appropriately. The fact that such a separating equilibrium exists and thus a firm can always use pricing to signal its availability is somewhat surprising, given that it does not depend in any way on the magnitude of the difference in inventory. This separation is facilitated by the fact that the use of prices to signal availability lends an implicit credibility to the signal as it may make imitating other firm types potentially quite expensive.

4.2. Pooling equilibrium

A pooling equilibrium is characterized by a single price p_{pool} that is selected by both firm types in equilibrium and beliefs $\theta_{pool}(\cdot) \in [0,1]$ with $\theta_{pool}(p_{pool}) = \bar{\theta}$, such that neither firm-type has any incentive to deviate. That is, for all prices $p \geq \underline{v}$, we must have:

$$R_h(p, \theta_{pool}) \leq R_h(p_{pool}, \theta_{pool}) \tag{9}$$

$$R_l(p, \theta_{pool}) \leq R_l(p_{pool}, \theta_{pool}). \tag{10}$$

Proposition 2. *A pooling equilibrium exists and survives the intuitive criterion only if a costly separating equilibrium exists and each firm type prefers their pooling revenue compared to their equilibrium payoff in the costly separating equilibrium.*

Proof. Suppose that a costless separating equilibrium and a pooling equilibrium with a period 1 price of p_{pool} and beliefs θ_{pool} exist for the same set of parameters. Notice that the h-type firm weakly prefers its revenue in the pooling equilibrium to its full information revenue, which it prefers compared to pricing at p_l^{fi} and being perceived as the l-type firm. Further, the l-type firm prefers to price at p_l^{fi} if it is identified as the l-type compared to any other price at any other belief. Thus, applying the intuitive criterion, it follows that if the customers observe an out-of-equilibrium price of p_l^{fi}, they must believe that it is the l-type firm, and hence the pooling equilibrium does not survive the intuitive criterion. So, it follows that a pooling equilibrium can only exist if there is no costless separating equilibrium for the same set of parameters, or equivalently, if there is a costly separating equilibrium.

Consider now the case in which a costly separating equilibrium exists. If the h-type firm prefers its costly separating equilibrium payoff to the pooling equilibrium, then noting that in the costly separating equilibrium the h-type firm is identified as the h-type firm, it follows that the h-type would in fact deviate and the pooling equilibrium cannot exist. Suppose instead that the l-type firm prefers its costly separating equilibrium revenue to the pooling revenue, and the h-type firm prefers its revenue in the pooling equilibrium to that in the costly separating equilibrium. Note that the h-type firm obtains its full information revenue in the costly separating equilibrium and prefers it to pricing at p_l' (which is the l-type firm's price in the costly separating equilibrium) and being perceived to be the l-type firm. It thus follows that the h-type firm prefers its pooling equilibrium revenue to that obtained by pricing at p_l' and being perceived to be the l-type firm. Then, the intuitive criterion implies that an out-of-equilibrium price of p_l' must then be associated with the belief that the firm is of l-type, and hence the pooling equilibrium does not survive the intuitive criterion. *Q.E.D.*

As is expected in signaling games, we find the existence of a pooling equilibrium. However, a pooling equilibrium can exist only for parameters for which a costless separating equilibrium does not exist. Intuitively, if a costless separating equilibrium exists, then it implies that both firm types prefer their full information prices and revenues to imitating the other type and so there is no reason for the firm types to pool. If a costless separating equilibrium does not exist, i.e., we have costly separation, then the l-type firm prices higher than its full information and loses revenue compared to it. Proposition 2 proves that this loss in revenue is, in fact, so severe that when a pooling equilibrium exists it Pareto dominates the corresponding costly separating equilibrium, i.e., both firm types have a higher revenue in the pooling equilibrium in contrast with the separating equilibrium.

5. The cost of signaling availability

In this section, we characterize how the cost of signaling availability depends on the relation between the firm types, in particular, we would like to identify when is separation costly and when is it costless.

PROPOSITION 3. *For any discrete demand distribution, if under full information and the set of prices* $\{p_h^{fi}, p_l^{fi}\}$*, stock-outs occur with positive probability and depend only on the demand realization and not on the firm type nor the price, then there can only be a costly separating equilibrium.*

Proof. If stock-outs depend only on the demand realization and not on the firm type, we cannot have stock-outs in the first period. This is so because if the l-type firm runs out of inventory in the first period, it must be the case that the demand in the first period precisely equals its inventory so that it this price the h-type firm would not stock-out if the customers believe the firm to be l-type.

For a separating equilibrium at the full information prices (p_h^{fi}, p_l^{fi}), the non-mimicry conditions must hold for each firm type. In particular, each firm type must prefer to price at its full information level and have its type identified as opposed to mimicking the other type. This condition can be expressed for the h-type firm as follows:

$$\begin{aligned}&(p_l^{fi}-s)(D_s\bar{F}(m(p_l^{fi},l))+D_m\bar{F}(p_l^{fi}))+sE[\min(D_s+D_2+D_m\bar{F}(p_l^{fi}),Q_h)]\\&\le (p_h^{fi}-s)(D_s\bar{F}(m(p_h^{fi},h))+D_m\bar{F}(p_h^{fi}))+sE[\min(D_s+D_2+D_m\bar{F}(p_h^{fi}),Q_h)],\end{aligned}$$

and for the l-type firm as follows:

$$\begin{aligned}&(p_l^{fi}-s)(D_s\bar{F}(m(p_l^{fi},l))+D_m\bar{F}(p_l^{fi}))+sE[\min(D_s+D_2+D_m\bar{F}(p_l^{fi}),Q_l)]\\&> (p_h^{fi}-s)(D_s\bar{F}(m(p_h^{fi},h))+D_m\bar{F}(p_h^{fi}))+sE[\min(D_s+D_2+D_m\bar{F}(p_h^{fi}),Q_l)],\end{aligned}$$

where the strict inequality follows because p_l^{fi} is the unique optimizer of l-type firm's full information solution.

Let $\mathcal{S}$ represent the realizations of S on which both firms stock out. Then, the non-mimicry condition for the h-type firm simplifies to

$$\begin{aligned}&(p_l^{fi}-s)(D_s\bar{F}(m(p_l^{fi},l))+D_m\bar{F}(p_l^{fi}))+sE[D_m(\bar{F}(p_l^{fi})-\bar{F}(p_h^{fi})),S\in\mathcal{S}^c]\\&\le (p_h^{fi}-s)(D_s\bar{F}(m(p_h^{fi},h))+D_m\bar{F}(p_h^{fi})),\end{aligned}\tag{11}$$

and that for the l-type firm simplifies to:

$$(p_l^{fi}-s)(D_s\bar{F}(m(p_l^{fi},l))+D_m\bar{F}(p_l^{fi}))+sE[D_m(\bar{F}(p_l^{fi})-\bar{F}(p_h^{fi})),S\in\mathcal{S}^c]\tag{12}$$

$$> (p_h^{fi} - s)(D_s\bar{F}(m(p_h^{fi}, h)) + D_m\bar{F}(p_h^{fi})).$$

Thus, (11) and (12) cannot hold simultaneously and we cannot have a costless separating equilibrium. *Q.E.D.*

This result states that when firms are "similar" in the sense that they stock-out for the same demand realizations, even though their corresponding availability levels might be different, we can only obtain costly separation. This result has implications for the cost of signaling. In particular, if the firms have a "coarse" understanding of the underlying demand, i.e., demand is forecasted only using a few scenarios, then it would be easier for the h-type firm to mimic the l-type and claim a lower availability than under full information. This propensity to mimic pushes the prices of the l-type firm higher and makes it costly for it to signal its true availability. However, if the demand forecast is made more sophisticated so that given the inventory differential between the two firms, it is not possible to sustain a similar availability risk in all demand scenarios, then the cost of signaling should decrease, and it may even drop to zero, i.e., we may obtain costless separation. A recent survey conducted by Bain and Co. showed that different industries use varying levels of coarseness in their demand forecasting. For example, the consumer packaged good uses a "single number" type of forecasts, the technology manufacturing uses a more refined "multiple scenarios," while the bio-pharma and the automotive industry further refine their forecasts and may use a "full distribution." Our results thus suggest that signaling via prices may work differently across industries.

Proposition 3 can also be viewed as stating that if the firm types are similar in terms of how they view the market, i.e., when do they run out of stock and when do they not run out of stock, then we can only obtain costly separation. We next consider the alternative view point, that of the customers which is the firm's inventory. In particular, we study the "ease" of separation as the inventory of the h-type firm increases. Noting that the myopic customers align each firm type with its quantity, i.e., the smaller the number of myopic customers needed to separate, the easier it is for the firm types to separate, we use the number of myopic customers as a proxy for the ease of separation. Figure 1 demonstrates the threshold minimum number of myopic customers required for costless separation as a function of Q_h for a scenario with $Q_l = 5$, $D_s = 5$, $F \sim U[1,3]$ and D_2 takes values 5 or 10, each with probability 0.5. This figure demonstrates that it is easier for the firm types to separate when their inventory levels are either very different or close to each other. It is in fact when their inventory differential is moderate that it is the most difficult to separate. So, we observe that when from the customers' perspective the firms are similar we obtain costless separation.

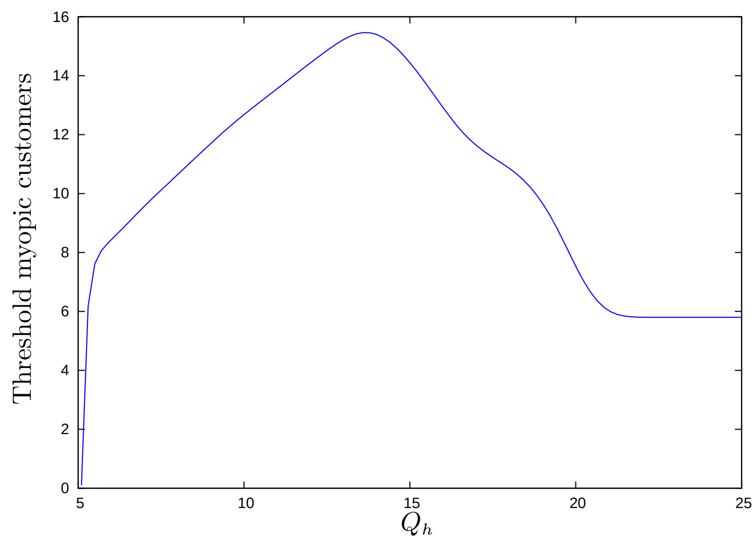

Figure 1 **The minimum number of myopic customers needed for costless separation as a function of the h-firm type's quantity.**

6. Discussion

We now discuss how some of the assumptions made in the model can be relaxed without affecting the results.

Optimal inventory decision. Consider the case in which the firm is not endowed with an inventory but rather selects it at the beginning of the game in an optimal fashion. In this case, the firm types are determined by the per unit costs of purchasing inventory and any other associated costs, such as holding costs. Consider any Perfect Bayesian Nash equilibrium of the corresponding game. One would expect the two firm types would stock different quantities because of their different underlying costs, and let us denote these quantities by Q_h^* and Q_l^*. Noting the sub-game remains the same as that considered in the previous sections, all the results apply unchanged.

Inventory information is public but demand information is private. In this case, we denote the firm's inventory level by Q, which is public. However, the demand is stochastic and can be of one of two types. In particular, in one state of the world, the firm has demand (D_m, D_s, D_2) and is labeled l-type, whereas in the other state of the world the firm has demand $(\beta D_m, \beta D_s, \beta D_2)$, where β is a constant less than 1 and is labeled h-type. We can translate this set up into that described in the previous sections by noting that the h-type firm's optimization problem and availability levels are unaffected if we scale down its inventory level and demand level by β, i.e., we set $Q_h := Q/\beta$, and the demand is now equal to (D_m, D_s, D_2). Setting $Q_l = Q$, we recover our initial model.

Optimizing the second period price. The model in the paper assumes that the second period price is set equal to a clearance price, which equals the lowest customer valuation. A more general setting would be one in which the firm may set the second period price after observing the sales in the first period.[3] While in this setting the firm sets prices in both periods, the signaling game as before occurs only in the first period. Indeed in the second period, the customers do not have the option to postpone their purchase. While this game is similar to that with clearance pricing, in this case, strategic customers make their purchase decisions based on the period 2 price risk in addition to the availability risk of the second period, which adds significant complexity to the analysis.

We test the robustness of our results in this setting by performing a numerical experiment using the same parameters as those in the previous section while optimizing over the second period price. We find that as in the model with clearance pricing, a separating equilibrium always exists and further we find the presence of both costly and costless separating equilibria. Figure 2 plots an analog of Figure 1, depicting the threshold number of myopic customers required for costless separation as a function of the quantity of the h-type firm when the firm optimizes the period 2 price. There are two observations one can make from this figure. First, the trend of the minimum number of myopic customers needed for costless separation is the same as in the model with price commitment, and second, compared with the model with commitment, fewer myopic customers

[3] Alternatively, the firm could commit to a second period price before observing the sales of the first period. Investigating such a model would be an interesting endeavor that we leave for a future study.

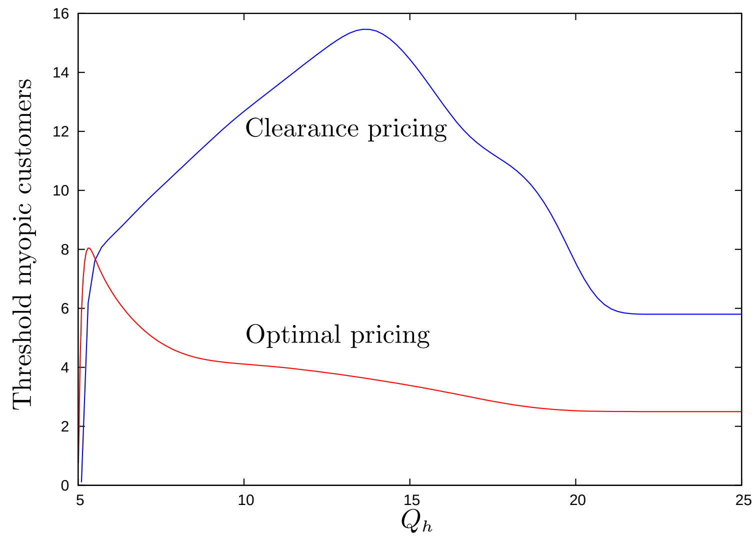

Figure 2 **The minimum number of myopic customers needed for costless separation as a function of the h-firm type's quantity for the cases of clearance pricing in period 2 and optimal pricing in period 2.**

seem to be needed for costless separation. The latter suggests that the lack of price commitment makes it "easier" for the low-quantity firm to signal its scarcity. A detailed analysis of this model is left as a topic for future study.

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Appendix. Supporting Result

Lemma 1. *If $g(x)$ is a non-increasing function, and $f(x)$ and $f(x)+g(x)$ are quasi-concave functions, then $h(x)\equiv\min\{f(x),b\}+\min\{g(x),b\}$ is quasi-concave.*

Proof. Given the fact that $f(x)+g(x)$ is quasi-concave, there exists $x_1 \in [0,\infty]$ such that $f(x)+g(x)$ is non-decreasing on $[0,x_1]$ and non-increasing $[x_1,\infty)$. Define $x_f := \inf\{x \geq 0 : f(x) = b\}$ and $x_g := \sup\{x \geq 0 : g(x) \geq b\}$. We consider two cases:

Case I: $g(0) < b$**.** We obtain that $h(x)$ is non-decreasing on $[0, \min\{x1, x_f\}]$ and non-increasing on $[\min\{x1, x_f\}, \infty)$. This completes the proof.

Case II: $g(0) \geq b$**.** We obtain that $h(x)$ is non-decreasing on $[0, \min\{x_1, x_f, x_g\}]$. If $x_1 \leq \min\{x_f, x_g\}$, then it follows that $h(x)$ is non-decreasing on $[0, x_1]$ and non-increasing on $[x_1, \infty)$. If $x_f \leq \min\{x_1, x_g\}$, then it follows that $h(x)$ is non-decreasing on $[0, x_f]$ and non-increasing on $[x_f, \infty)$. If $x_g \leq \min\{x_1, x_f\}$, then if $x_f \leq x_1$, then it follows that $h(x)$ is non-decreasing on $[0, x_f]$ and non-increasing on $[x_f, \infty)$, and finally if $x_f > x_1$, then it follows that $h(x)$ is non-decreasing on $[0, x_1]$ and non-increasing on $[x_1, \infty)$. This completes the proof. *Q.E.D.*